

Fig. 2 MHD-driven circulatory flow between stationary cylinders ($e_z = 0.5$, $R = 0$, $C = 1.0$, $\kappa = 0.25$): a) $Q = 1.0$, $v_m = 0.405$; b) $Q = 10$, $v_m = 1.77$; c) $Q = 100$, $v_m = 2.94$; d) $Q = 1000$, $v_m = 3.60$.

in Figs. 1 through 3. In the figures, the velocity has been normalized by dividing by its maximum value v_m :

$$v_m = \max_{\kappa \leq \xi \leq 1} v_\theta \quad (5)$$

where κ is the ratio of inner to outer cylinder radius.

When Q is large (e.g., $Q = 1000$ in the figures), the velocity profiles approach the curve

$$v_\theta = (C'/\xi) + G\xi \quad (6)$$

in the interior of the fluid, where C' and G are determined by the applied fields and are independent of the velocities of the confining cylinders. Boundary layers are formed near the cylinders. The effect of the imposed radial flow is to increase the thickness of the boundary layer near the injection cylinder and to decrease the thickness of the boundary layer near the suction cylinder.

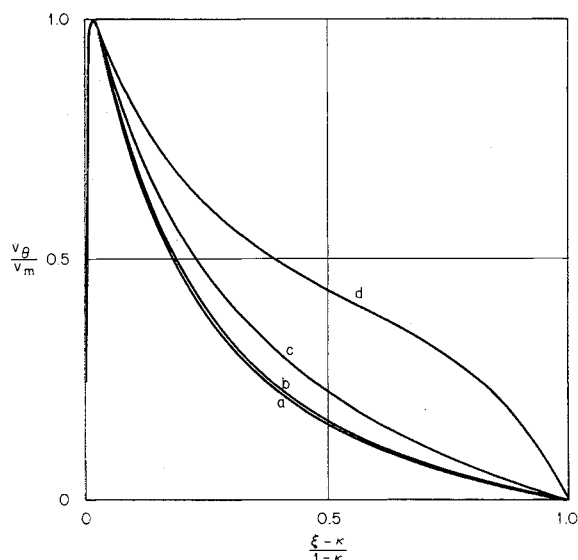


Fig. 3 MHD-driven circulatory flow between stationary cylinders ($e_z = 0.5$, $R = 100$, $C = 1.0$, $\kappa = 0.25$): a) $Q = 1.0$, $v_m = 0.016$; b) $Q = 10$, $v_m = 0.152$; c) $Q = 100$, $v_m = 1.06$; d) $Q = 1000$, $v_m = 2.92$.

There are a few minor misprints in Ref. 2: 1) Equation at the bottom of page 380 should be

$$-C \frac{d}{dr} (ru_\theta) - \frac{\mu_e D}{4\pi\rho} \frac{d}{dr} (rH_\theta) = \nu r^2 \frac{d}{dr} \left(\frac{du_\theta}{dr} + \frac{u_\theta}{r} \right)$$

2) μ_2 in Eq. (11) should be μ_e ; 3) $\eta_{1,2}$ in Eq. (16) should be $n_{1,2}$; 4) η_1 in Eq. (23) should be n_1 ; 5) B in Eq. (29) should be B_1 ; and 6) A in Eq. (31) should be A_1 .

References

- ¹ Chang, T. S., "Radially Symmetric Motion of a Conducting Fluid Under the Influence of Radial and Transverse Magnetic Fields," AEC-ORNL-PR-1-12, Aug. 1965, unpublished Consultant Report, Oak Ridge National Laboratory, Oak Ridge, Tenn.
- ² Gupta, A. S., "Circulatory Flow of a Conducting Liquid About a Porous, Rotating Cylinder in a Radial Magnetic Field," *AIAA Journal*, Vol. 5, No. 2, Feb. 1967, pp. 380-382.
- ³ Lewellen, W. S., "Magnetohydrodynamically Driven Vortices," *Proceedings of the Heat Transfer and Fluid Mechanics Institute*, 1960, pp. 1-15.

Addendum: "A Theoretical Investigation of MHD Channel Entrance Flows"

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EQUATION (5) of our paper was an asymptotic expression for the fully developed friction factor for turbulent MHD flow between parallel plates. The exponent of the denominator of the right-hand side of this equation should

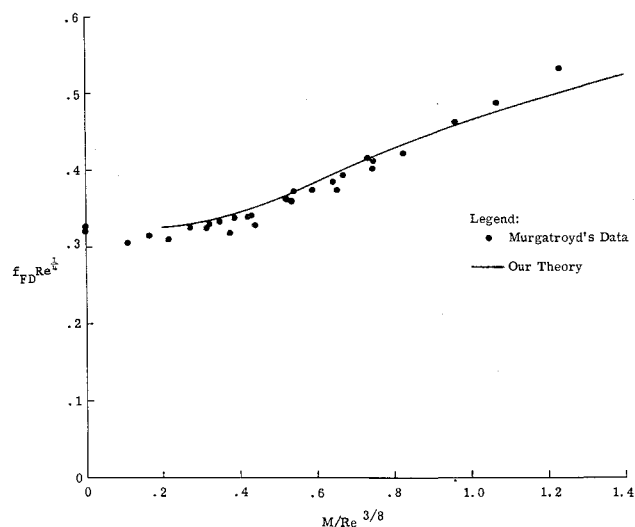


Fig. 1 Comparison of new theoretical expression [Eq. (2)] with experiment.

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be $\frac{8}{5}$ instead of $\frac{7}{4}$; thus

$$f_{FD} = 0.446 \frac{(M/Re)^{2/5}}{\{1 - [0.0139/(M/Re)^{8/5}Re]\}^{8/5}} \quad (1)$$

The effect of this correction is to improve the already good agreement with the experimental results of Murgatroyd, as shown in Fig. 7 of our paper.

Recently, we noticed that the corrected Eq. (1) can be written in a more informative manner in which the total number of variables is reduced from three to two:

$$f_{FD}Re^{1/4} = \frac{0.446 (M/Re^{3/8})^{2/5}}{\{1 - [0.0139/(M/Re^{3/8})^{8/5}]\}^{8/5}} \quad (2)$$

This equation shows that the quantity $f_{FD}Re^{1/4}$ is a function only of the independent variable $M/Re^{3/8}$. In Fig. 1, our theory as given by Eq. (2) is compared to the experimental

results of Murgatroyd. Agreement of theory with experiment is seen to be quite close. As described in our paper, it is not possible to extend our theory down to zero Hartmann number (zero abscissa).

Equation (2) indicates the fact that fluid viscosity, while being a weak effect, is still important in the turbulence mechanism. It is interesting to note the close similarity of the independent variable $M/Re^{3/8}$ to $M/Re^{1/2}$, which is the square root of the ratio of magnetic to inertial forces. The latter dimensionless ratio, namely M^2/Re , contains no viscosity and would be expected to be an important variable for completely turbulent flow, such as flow past a very rough wall. Samaras¹ has discussed the importance of this so-called Cowling -1 number.

Reference

- ¹ Samaras, D. G., *Theory of Ion Flow Dynamics*, Prentice-Hall, Englewood Cliffs, N.J., 1962, pp. 367, 416.